Abstract

The following thesis studies several probabilistic models related to the distributional equation of the form

$$X \stackrel{d}{=} \sum_{k=1}^{N} A_k X_k + B, \tag{2}$$

where $(B, N, (A_k)_{k\in\mathbb{N}}) \in \mathbb{R}\times\mathbb{N}\times\mathbb{R}^{\mathbb{N}}$ is a random vector independent of the sequence $X, X_1, X_2, \ldots$ of independent identically distributed random variables with a common law μ. We assume that the distribution of the vector $(B, N, (A_k)_{k\in\mathbb{N}})$ is given and aim to study μ for which (2) holds.

The first part of the thesis concerns a description of the tail asymptotic of X. In the first chapter we treat the case of so-called random difference equation, i.e. $N = 1$, with (A_1, B) possessing a regularly varying distribution. Second chapter is devoted to the equation of the form (2) appearing during probabilistic analysis of a class of "divide and conquer" algorithms. The common feature of such equation is that A_k's are nonnegative, B is bounded and $\sum_{k=1}^{N} A_k = 1$.

The second part is dedicated to processes in random medium which are related to (2). In the third chapter we study a branching process in random environment. More precisely a version of a classical Galton-Watson process where the reproduction law i picked at random at each generation. The last chapter is devoted to a random walk in random environment. That is a nearest neighbour random walk on $\mathbb{Z}$ where the transition probabilities have been picked at random. Using relation with (2) we investigate large deviations for both processes.