

Abstract

In this thesis we report theoretical results on the asymptotic FDR control by the SLOPE procedure under Gaussian design for the Linear and the Logistic Regression. In the first step we provide a general and exact formula for FDR of SLOPE for any convex loss function, which is expressed in terms of the distributional properties of the gradient of this function at the SLOPE estimate (see Theorem 3.4). Then we discuss the Linear Regression under the classical set-up when p is fixed and n diverges to infinity. Here we prove the root- n consistency of SLOPE estimators and the asymptotic FDR control (see Theorems 3.10, 3.12). We also provide conditions on the signal magnitude under which the power of SLOPE is close to 1. Then we discuss the high dimensional setup, where p can diverge to infinity much quicker than n . Here we show that the asymptotic FDR control by SLOPE is possible if the number $k(n)$ of nonzero elements in the true vector of regression coefficients satisfies $k = o\left(\sqrt{\frac{n}{\log p}}\right)$ and values of these non-zero elements are sufficiently large, so that the asymptotic power of SLOPE converges to 1. Furthermore we show that when k is bounded the assumption on the signal strength is redundant and SLOPE can control FDR even when the signals are at the noise level (see Theorem 3.30). Finally we discuss the Logistic Regression in the low dimensional set-up. We prove the root- n consistency of SLOPE estimators. Moreover, we show the asymptotic FDR control in case when the signal magnitude converges to 0 at an almost arbitrary rate. The results cover the case of contiguous alternatives, which are the main target of the SLOPE procedure (see Theorems 3.47, 3.50).