Abstract

This doctoral dissertation focuses on aggregation-diffusion equations (ADE), which model the behavior of particle density under non-local interactions and nonlinear diffusion. These equations describe various physical and biological phenomena, including, e.g., gravitational attraction, chemotaxis and swarming. Modeling through ADE captures long-range attraction and short-range repulsion, serving as a continuum description of particle interactions derived from ordinary differential equations. This dissertation is divided into three parts.

In the first part, an L^p-theory is developed for an ADE with a power-law interaction kernel. Global-in-time and globally bounded solutions exhibit a concentration phenomenon for small diffusion coefficients, resulting in significant mass accumulation in small neighborhoods around the origin. This can be viewed as a qualitative description of the singularity that forms in the absence of diffusion. The main calculations are based on *a priori* and moment estimates, and time averaging through integration. Properties of the solutions for large times are also described.

The second part is devoted to discussing the application of fixed-point methods to establish the existence of steady states for certain ADE with a more general power-law kernel. Moreover, explicit formulas for some stationary solutions are derived, which are valuable for verifying general numerical methods. Finally, the existence of a sign-changing solution in one dimension is demonstrated, utilizing results from the theory of the Burgers' equation.

The last part analyzes a minimal parabolic-elliptic Keller-Segel system modeling cell density and interactions through chemoattractant concentration. In this system, constant functions serve as steady states. A framework is developed for local-in-time solutions in the uniformly local Lebesgue spaces, alongside an analysis of the long-term dynamics of these solutions. Certain constant stationary solutions are stable, indicating that small perturbations can lead to global-in-time convergence. Conversely, beyond a critical parameter value, these constant steady states exhibit instability.