NON-CONVENTIONAL, NON-LINEAR, NON-LOCAL EVOLUTION PDEs: A TWO-CASE STUDY

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ABSTRACT. This thesis consists of the study of two largely independent problems, both of which fall into the category of evolution partial differential equations. Both are at the same time non-conventional in the manner they are posed: one because of the set of initial conditions, the other because of an untypical class of "differential" operators involved.

In the first part we consider a non-local evolution equation generalising the viscous Burgers equation supplemented by an initial condition which is a homogeneous random field. We develop a non-linear framework enabling us to show the existence and regularity of solutions as well as study their long time behaviour.

In the second part we consider the Cauchy problem for a general non-local evolution equation which includes i.a. some cases of the porous medium equation with the fractional Laplacian and the parabolic equation with the fractional p-Laplacian. We show the existence, uniqueness and other fundamental properties of solutions and we give new examples of non-local, non-linear operators which exhibit interesting behaviour.