Abstract

The dissertation focuses on problems related to Markov chains on finite state spaces, which can be solved using certain dualities between the chains. In general, we say that the chain $\mathbf{X}^*$ with the transition matrix $\mathbf{P}_{X^*}$ is dual to the chain $\mathbf{X}$ with the transition matrix $\mathbf{P}_X$ if (we omit the conditions for initial distributions)

$$\Lambda \mathbf{P}_X = \mathbf{P}_{X^*}\Lambda,$$

where Λ matrix is a so-called *link*. Different links define different dualities. For specific types of duality, additional assumptions on transition matrices are needed. These assumptions are closely related to the ordering, most often partial, of the state space.

In the first chapter of the dissertation, we consider the Möbius monotonicity, its relations to other monotonicities (including, among others, stochastic monotonicity) and with *perfect simulation* – a method that returns an unbiased sample from the stationary distribution of a given ergodic Markov chain. In this chapter, we present a new algorithm for this perfect simulation based on the so-called *strong stationary duality*, its use requires the aforementioned Möbius monotonicity.

The next chapter is devoted to multidimensional gambler's ruin models. It turns out that the Kronecker products *work* well with matrix formulas for duality, which allows to generalize the classical results. In particular, we show a large family of multidimensional chains (which correspond to some multidimensional versions of the gambler's ruin model) that have the same probability of winning (we provide the exact formula) and/or have the same distribution of time till win/lose (we provide its structure).

The last chapter is related to the distribution of the game duration in the gambler's ruin model (one-dimensional) conditioned on the event of winning or losing – we consider model with arbitrary winning/losing probabilities in one step. We show, among others, interesting symmetries of the results for the original model and the model with swapped winning/losing probabilities. Using the obtained results (and strong stationary duality), we improve the results on the speed of convergence to stationarity for the symmetric random walk on a circle.

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