

Groups and rings in some model-theoretic and model-theory-motivated contexts

Jan Dobrowolski

Supervisor: dr hab. Krzysztof Krupiński

Abstract

The thesis contains some new results about groups and rings in various contexts.

In Section 1, we prove that every ω -categorical, generically stable group is nilpotent-by-finite, and that every ω -categorical, generically stable ring is nilpotent-by-finite. These theorems generalize results of Baur-Cherlin-Macintyre and Felgner (in the case of groups) and of Baldwin-Rose (in the case of rings).

In Section 2, we prove some structural results about (weakly) locally finite profinite rings. The main results are: a complete classification of semisimple (weakly) locally finite profinite rings, and a theorem stating that the Jacobson radical of every locally finite profinite ring is nil of finite nil exponent. These results apply in particular to the class of small compact G -rings, yielding generalizations of certain results obtained by Krupiński and Wagner for small profinite rings (in the sense of Newelski).

In Section 3, we present certain constructions of examples of small Polish group structures which solve two problems stated by Krupiński. First of them is the construction of the first known non-zero dimensional small Polish G -group, and the second yields examples of small Polish group structures without nm -generic orbits.

In Section 4, we introduce some canonical topologies induced by actions of topological groups on groups and rings. For H being a group [or a ring] and G a topological group acting on H as automorphisms, we describe the finest group [ring] topology on H under which the action of G on H is continuous. We also study the topologies that we introduced in the context of Polish structures. In particular, we prove that there may be no Hausdorff topology on a group H under which a given action of a Polish group on H is continuous.