We look for nontrivial solutions to the semilinear problem

$$\nabla \times \nabla \times u = f(x, u) \text{ in } \mathbb{R}^3,$$

where $u : \mathbb{R}^3 \to \mathbb{R}^3$ and $f = \nabla_u F$. We prove the existence of a ground state solution and of infinitely many geometrically distinct solutions.

After building the proper function space where to look for solutions, we develop an abstract critical point theory which, once back in the concrete setting, somehow allows us to work only in the subspace of divergence free vector fields. Later on, we apply the results from the astract part to our case.

The main difficulties are due to working in an unbounded domain and the infinite dimension of the kernel of $u \mapsto \nabla \times u$, i.e. the space of gradient vector fields. A further, yet not structural, difficulty is given by the "little restrictive" growth hypotheses on the nonlinearity, i.e. we do not necessarily assume a power type or a double-power type behaviour, but provide them using a more or less generic N-function Φ.

This talk is based on a work in progress with Jarosław Mederski and Andrzej Szulkin.