

Let X_1, X_2, \dots, X_n denote i.i.d. centered standard normal random variables, then the law of the sample variance $Q_n = \sum_{i=1}^n (X_i - \bar{X})^2$ is the χ^2 -distribution with $n - 1$ degrees of freedom. It is an open problem in classical probability to characterize all distributions with this property and in particular, whether it characterizes the normal law. Now we present a solution of the free analogue of this question and show that the only distributions, whose free sample variance is distributed according to a free χ^2 -distribution, are the semicircle law and more generally so-called *odd* laws, by which we mean laws with vanishing higher order even cumulants. In the way of proof we derive an explicit formula for the free cumulants of Q_n which shows that indeed the odd cumulants do not contribute and which exhibits an interesting connection to the concept of preservation of free infinite divisibility.